Maxima for teachers and other busy humans

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Life of Larry, Math Teacher

Let's follow a typical Monday evening for Larry, Math Teacher:

- Larry is a twenty-two-year-old 2014 UNK graduate.
- He teaches 10th grade algebra and pre-calculus at Broken Knuckle High, Broken Knuckle, Nebraska.
- Although (occasionally) brilliant, Larry frequently makes mistakes and usually procrastinates.
- The banal and mundane bore Larry–when he is bored, he daydreams and makes *many* mistakes.

Data: cornbread, mac 'n cheese, mashed potatoes, apple pie
Result: temporary satisfaction
unsatisfied ← *true*
while *unsatisfied* **do**
  enjoy;
end

- Larry's eating habits need an immediate and effective intervention.
- Maxima *cannot* help with that.

Larry has three stacks of papers to grade:

- Before grading (not halfway through), is a good time to write a key.
- Should Larry check his key? Maxima can help with that:

```
(%i1) %solve(abs(x-7)=11-x/2,x);
```

(%o1) $\%\mathrm{union}\left([x=-8],[x=12]\right)$

```
(%i2) %solve(cos(2*x)=sin(x),x);
```

(%o2) $\%\mathrm{union}\left([x=-\frac{4\pi\%z172+\pi}{2}],[x=\frac{4\pi\%z173+\pi}{6}]\right)$

```
(%i3) factor(x^6-1);
```

(%o3) $(x-1)\,(x+1)\,\left(x^2-x+1\right)\,\left(x^2+x+1\right)$

```
(%i4) 500.0 * exp(1.03);
```

(%o4) 1400.53291734954

8:47 PM – Daydreaming

Larry needs to sum the points on a 42 point exam and scale to 100. It's boring work, so Larry is daydreaming. Maxima can help:

```
(%i1) g([l]) := block([x : xreduce("+",l)],
                          [x, round(100*x/42)])$
```

On pages one through four, Sally's scores are 9, 7, 5, and 12 points; so

```
(%i2) g(9,7,5,12);
```

(%o2) $[33, 79]$

- With just a few more lines of code, Larry's function could fudge the grades using a weird function (not that Larry would do such a thing).
- Warning: Dr. Nebesniak tells me that if a paper has score at the top of the exam, it's likely students will not look at anything else.

Larry needs to check his class notes for Tuesday. Maxima can help find a step-by-step solution set to $|x-7| = 11 - x/2$.

```
(%i1) eq : abs(x-7)=11-x/2;
```

(%o1) $$|x-7| = 11 - \frac{x}{2}$$

```
(%i2) eq^2;
```

(%o2) $$(x-7)^2 = \left(11 - \frac{x}{2}\right)^2$$

```
(%i3) rhs(%)-lhs(%);
```

(%o3) $$\left(11 - \frac{x}{2}\right)^2 - (x-7)^2$$

```
(%i4) factor(%);
```

(%o4) $$-\frac{3\,(x-12)\,(x+8)}{4}$$

The rest is easy enough with carbon-based computing.

10:03 PM — Oops

Oops–Larry squared the equation, so he may have generated spurious solutions; gotta check:

```
(%i1) eq : abs(x-7) = 11-x/2$
(%i2) subst(x=12,eq);
```

(%o2) $5 = 5$

```
(%i3) subst(x=-8,eq);
```

(%o3) $15 = 15$

11:04 PM Oops redux

- After a few months of teaching, Larry already knows that students are disinclined to check their work.
- As an alternative to square and check, Larry uses pattern matching to append a rule to his equation solver.

```
(%i1) matchdeclare([a,b], lambda([q],true),
          x, symbolp)$
(%i2) tellsimpafter(Solveme(abs(a)=b,x),
      fourier_elim([a=b, b>=0] or [-a=b, b>=0],[x]))$
(%i3) Solveme(abs(x-7)=11-x/2,x);
```

(%o3) $[x = -8] or [x = 12]$

- Larry's `Solveme` function can be extended using pattern matching.
- The heavy lifting is done by `fourier_elim`.

10:47 PM — Tuesday's quiz

For Tuesday's quiz, Larry would like to generate an equation with a "nice" solution; Maxima can help:

```
(%i1) x*sqrt(x+1);
```

(%o1) $x\sqrt{x+1}$

```
(%i2) % = subst(x=8,%);
```

(%o2) $x\sqrt{x+1} = 24$

Further he can typeset it in TeX for pasting into a document:

```
(%i3) tex(%);
```

```
$$x\,\sqrt{x+1}=24$$
```

And he can check if there are more solutions:

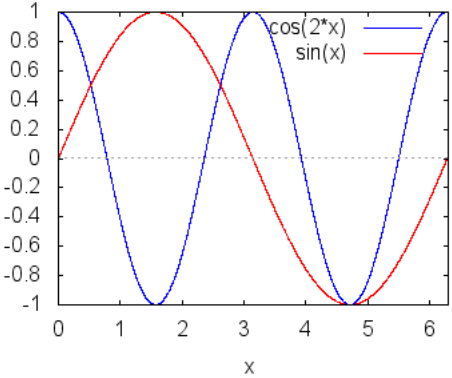
```
(%i4) %solve(x*sqrt(x+1)=24,x);
```

(%o4) $\%\mathrm{union}\left([x = 8]\right)$

9:51 PM A picture is worth ten points

Tuesday's quiz needs a graph. In Maxima, Larry can create a graph, right click the graphic, save it as a png file, and include it in a (what else) $\TeX$ (or a Word[1]) file.

```
(%i2) wxplot2d([cos(2*x),sin(x)],[x,0,2*%pi]);
```

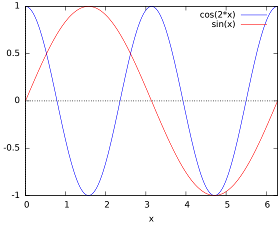


[1] ☹

pdf graphs

For a higher quality graph, Larry can use pdf output from `gnuplot`:

```
(%i10)plot2d([cos(2*x),sin(x)], [x,0,2*%pi],
        [gnuplot_term, pdf],
        [gnuplot_out_file, "sine-cosine-graph.pdf"])$
```



To paste into the TEX document, use

```
\begin{figure}[p]
    \includegraphics[scale=0.4]{sine-cosine-graph.pdf}
\end{figure}
```

11:22 PM Guilty pleasures

To eliminate the banal and mundane, Larry indulges in his guilty pleasure: Maxima code for the Landau O symbol:

Looks great:

```
(%i1) bigoh(1+5*x-x^2,x,inf);
```

(%o1) $\mathrm{bigoh}\left(x^2, x, \infty\right)$

Could be better (simplify to $\mathrm{O}(x, x, \infty)$):

```
(%i2) bigoh(29*x,x,inf)-bigoh(2015*x,x,inf);
```

(%o2) $\mathrm{bigoh}\left(x, x, \infty\right) - \mathrm{bigoh}\left(x, x, \infty\right)$

- Larry enjoys seeing how much math can be stuffed into just 18 lines of Common Lisp. But ...
- the first ninety percent of most everything is easy ...

2:43 AM Attempt to fall asleep

Maxima *can* help with that: much of the Maxima user documentation serves as a paper sedative.

Don't despair–there are non soporific alternatives; see for example

- http://www.math.harvard.edu/computing/maxima/
- http://maxima.sourceforge.net/docs/tutorial/en/minimal-maxima.pdf
- http://www.csulb.edu/~woollett/

The Maxima sourceforge page has links to these documents.

$[\text{bw} \in \text{Teachers} \cup (\text{Busy} \cap \text{Human})] \equiv [\pi + e \in \mathbf{Q})]$

In bw five-valued logic,[2] where is the putative identity

$$\sum_{k=0}^{n} (-1)^k F(k) = \frac{F(0) + F(n)}{2} + \sum_{k=2}^{\infty} \left(F^{k-1}(n) - F^{k-1}(0)\right)(2^k - 1)\frac{B_k}{k!},$$

and[3]

$$B_n = \frac{1}{n+1} \sum_{k=1}^{n} \sum_{j=1}^{k} (-1)^j j^n \binom{n+1}{k-j} \Big/ \binom{n}{k}.$$

- Maxima verifies the identity whenever F is a polynomial of degree five or less and n is a positive even integer.

[2](i) manifestly bogus (*e.g.* $\pi > \infty$), (ii) bogus (*e.g.* $\pi < 3$), (iii) don't know (*e.g.* $\pi + e \in \mathbf{Q}$), (iv) conditionally nonbogus (*e.g.* $1 - (-1)^n = 0$), and (v) nonbogus (*e.g.* $\pi \geqslant 3$)

[3]TEX copied from the dlmf `http://dlmf.nist.gov/24.6.E2`.

Don't trust, but verify

```
(%i2) id:sum((-1)^k*f(k),k,0,n)=(f(n)+f(0))/2 +
      sum(bern(k)*(diff(f(n),n,k-1)-at(diff(f(x),x,k-1
```

(%o2) $$\sum_{k=0}^{n}(-1)^k\,\mathrm{f}(k)=$$

$$\left(\sum_{k=2}^{M}\frac{\left(2^k-1\right)\,\mathrm{bern}(k)\,\left(\frac{d^{k-1}}{d\,n^{k-1}}\,\mathrm{f}(n)-\left.\frac{d^{k-1}}{d\,x^{k-1}}\,\mathrm{f}(x)\right|_{x=0}\right)}{k!}\right)+\frac{\mathrm{f}(n)+\mathrm{f}(0)}{2}$$

```
(%i3) simplify_sum(id), f(x) := x^5, M : 6,
        diff,sum,at;
```

(%o3) $$\frac{(2\,n+1)\,\left(n^2+n-1\right)^2\,(-1)^n}{4}-\frac{1}{4}=\frac{n^5}{2}+\frac{5\,n^4}{4}-\frac{5\,n^2}{4}$$

```
(%i4) factor(rhs(%)-lhs(%));
```

(%o4) $$-\frac{(2\,n+1)\,\left(n^2+n-1\right)^2\,\left((-1)^n-1\right)}{4}$$

Did bw assume facts not in evidence (n even) in his derivation?

Just the Facts

Maxima

- is a computer algebra system (CAS).
- is about 47 years years old.
- is free and open source–to find it, search for "maxima download."[4]
- is available for Android, IOS, Linux, OS X, and Microsoft Windows.
- can solve equations, factor, graph, and much more.
- has its own programming language.
- works with TEX for typesetting documents.
- is the symbolic engine for Euler, a MatLab like numerical system.
- is included in Sage, a comprehensive mathematics software system.
- was downloaded 215,973 times by users in 198 countries in the past year (Spain, Japan, and United States account for the most downloads).

[4] As of February 2015, the Nissan Maxima® is not downloadable, so the search finds the CAS.

Math Club Coda[5]

- This hasn't been a tutorial, but we've seen that Maxima:
  - can solve algebraic and trigonometric equations,
  - factor and expand expressions,
  - draw graphs for pasting into a document,
  - typeset expressions in TEX,
  - has a programming language,
  - can be extended using pattern matching or Common Lisp.
- And we've seen that TEX can typeset
  - mathematics (many examples),
  - graphics (A picture is worth ten points),
  - algorithms (Larry's dinner).
- Should somebody should volunteer to give a talk on TEX?

[5]Merriam-Webster: "Something that serves to round out, conclude, or summarize and usually has its own interest."

What you don't say is as important as what you don't

Expunged items include:

- All examples of *Deus ex machina*.[6]
- Some examples involving the number 42.
- A so called proof that uses $-\sum_{k=1}^{\infty}(-1)^k k = 1 - 2 + 3 - 4 + \cdots = \frac{1}{4}$.
- Maxima derivations of advanced methods for annoying your teacher:

$$\frac{\mathrm{d}^k}{\mathrm{d}x^k}\sin(x) = \sin\left(x + \frac{\pi}{2}k\right), \quad k \in ???$$

$$\overbrace{\int \cdots \int}^{k \text{ times}} \sin(x)\, dx\, dx \cdots\, dx = \sin\left(x - \frac{\pi}{2}k\right), \quad k \in ???.$$

[6]Wikipedia: A plot device whereby a seemingly unsolvable problem is suddenly and abruptly resolved by the contrived and unexpected intervention of some new event, character, ability or object.