2017–2018 Rathe Scholarship Application

Application A complete application includes your name, mailing address, email, phone number, name of high school, anticipated major, essay of 300 words or less, and solutions (full or partial) to the following five mathematical puzzles. The topic of your essay is: *What motivates you to pursue a UNK degree in Mathematics, Mathematics 7-12 Education, Computer Science, or Physics?* When you are finished, mail your application to:

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If you prefer, email your entry to roachcs2@unk.edu. Entries are due on or before March 1, 2018. To earn the award, you must attend UNK, be a freshmen in fall 2018, and must have a declared major in either Mathematics, 7-12 Mathematics Education, Computer Science, or Physics.

Puzzle Instructions Please submit solutions to the problems below. Even partial solutions, if well presented, can help your case for deserving this scholarship. We are looking for your ability to present your mathematical ideas to others in a clear and concise way. Your solutions should include some exposition about the reasoning behind the main points of your computations. Write your solutions in a form that you might see in a text book. In a text book, the solution is presented, and it is explained so that a reader can follow it. Your solutions should strive for this too.

1. Bob likes to keep his cash in envelopes. He currently has $63. What is the fewest number of envelopes he could use to keep his cash and still be able to spend it on a whole dollar cost purchase by just selecting some of the envelopes to hand the cashier? How much is in each of the envelopes?

2. My friends and I are trying to split up some marbles. We tried to split them 3 ways and had 2 left over. When we tried to split them 5 ways, we had 3 left over. When we tried to split them 7 ways, we had 5 left over. We have fewer than 100 marbles. How many marbles do we have?

3. Let $a$ and $b$ be real numbers and suppose that the line $y = ax + b$ and the parabola $y = x^2$ intersect in *exactly one point*. What is the relation between $a$ and $b$? Specifically if $a = 2$, sketch the line and the parabola on the same graph.

4. Show algebraically that $2\sqrt{\sqrt{3} + 2} = \sqrt{2} + \sqrt{6}$.

5. A group of aliens has landed in your backyard. Although their language is different than yours, you have found that you might be able to understand their numbers. It seems that they use a place value system similar to our own, and you have deciphered the following correspondence of symbols to numbers: $\cdot = 0, \odot = 1, \ominus = 2, \oplus = 3, \ominus = 4, \oslash = 5, \odot = 6, \otimes = 7$, and $\oslash = 8$. Now, they’ve counted the bricks in your Lego set and you know there are 786 pieces which they write as $\ominus \otimes \oslash \odot$. What base system do they use?