A Day at the Races

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When I was an undergraduate, the *Pi Mu Epsilon Journal* published an article that made a fantastic claim; it said it was possible to place bets on a horse race in such a way that the bettor would receive a guaranteed payoff. The article was very clear on how to do this, but apparently the author never tested her or his theory at the racetrack. The article reminded me of a silly 1960s movie *Dear Bridget*, starring Jimmy Stewart, about a young boy who wrote love letters to Bridget Bardot and who used his astounding mathematical abilities to pick winning horses. As I remember the article, the analysis was something like the following.

In a race of n horses, you wager x_1 dollars on the first horse, x_2 dollars on the second, $\ldots$, and x_n dollars on the n-th horse. If the i-th horse wins, your payoff is $p_i x_i$ dollars, where p_i is the pay out for the i-th horse. Your total investment is $x_1 + x_2 + \cdots + x_n$, so you win $w_i = p_i x_i - (x_1 + x_2 + \cdots + x_n)$ dollars if the i-th horse wins. Recognizing that these are linear equations for $x_1, x_2, \ldots, x_n$ and setting $n = 3$ and $w_i = w$, the equations in matrix form are

$$\begin{pmatrix} p_1 - 1 & -1 & -1 \\ -1 & p_2 - 1 & -1 \\ -1 & -1 & p_3 - 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} w \\ w \\ w \end{pmatrix}.$$

(For a real horse race, n is about 10.) Regardless of which horse wins, you go home w dollars richer. The odds are set by the track, so all you need to do is decide how much you would like to win, set each w_i to this value, solve the linear equations, place your bets, relax, collect your earnings, pay the IRS, and retire in Tahiti. A handsome reward for solving a few linear equations.

As I remember, the only caveat mentioned by the author was that the odds can change after the last bet is placed, so the earnings aren't actually guaranteed; however, the author argued that as long as the amount of each bet is a small percentage of the total bets, the odds would not change drastically as a result of one bet.

For no reason other than nature's no free lunch (NFL) policy, something is wrong with the analysis. If the article's claim were true, all mathematicians except those that believe gambling is morally wrong would be retired or rich. The author could have saved herself or himself embarrassment by solving these equations using actual data. At the time the article was written, hand held calculators that could solve these equations did exist, but were expensive. Why didn't the author purchase a calculator and head to the racetrack? Actually, a calculator isn't needed to solve the equations; we'll show that no matter the value of n, they are easy to solve by hand.

Let's solve the equations and in doing so figure out what is wrong with the article's claim. Defining $M = x_1 + x_2 + \cdots + x_n$ and starting with $p_i x_i - M = w_i$, we have

$$x_i = \frac{M}{p_i} + \frac{w_i}{p_i}.$$

Summing over i gives

$$\begin{aligned} M &= \sum_i x_i = \sum_i \frac{M}{p_i} + \sum_i \frac{w_i}{p_i}, \\ &= M \sum_i \frac{1}{p_i} + \sum_i \frac{w_i}{p_i}. \end{aligned}$$

Combining the two terms involving M, we have

$$\left(1 - \sum_i \frac{1}{p_i}\right) M = \sum_i \frac{w_i}{p_i}.$$

The right hand side is positive; thus if $\sum_i 1/p_i > 1$, we have $M < 0$. Unless you own the racetrack, it isn't possible to make a negative bet. Surely racetrack owners are crafty enough to set the odds so that $\sum_i 1/p_i > 1$.

The next issue of the *Pi Mu Epsilon Journal* published a retraction. In short, either own a racetrack or keep your day job and stay away from betting on horses.

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