Problem of the Fortnight

Rat Race

A researcher puts together a long corridor partitioned into 4 rooms. There is an exit at the right end of the corridor. A rat is placed into the left most of the 4 rooms every minute. At the same time, the doors between the rooms open up and each rat in the maze immediately leaves the room it is in according to the following rules: if in room 1, it goes to room 2; if in room 2, it goes to room 1 or room 3 with a .5 chance for each; if in room 3 it goes to room 2 or room 4 with a .5 chance for each; finally, if in room 4, it leaves the maze and gets some cake. (The cake is a lie.)

When the researcher runs the experiment for several hours, he realizes the number of rats in each room stays relatively constant. I.e., the average number of rats in each room has hit an equilibrium. The question to you is, what is the long-run average number of rats in each room?

For an added little challenge of less difficulty than the original problem: knowing the average number of rats in each room, what is the average length of time that a rat is in the ‘maze’?

The Problem of the Week is open to all undergraduate students, regardless of major. Submit your written solution, along with your name and e-mail address, to the Math Department office (Founders Hall Room 2006) by 2:00 p.m. on Friday, February 19, 2016. The person who submits the best solution will win a $10.00 gift certificate to the Barista’s Daily Grind. There will be prizes for the 3 participants with the greatest number of correct solutions throughout the semester. So even if your solution isn’t ‘nice’, you should turn it in for a chance to add to your tally of correct solutions.

http://www.unk.edu/academics/math/problem-of-the-fortnight.php