Math Problem of the Fortnight

Center Court Solution

A game show has contestants tied together with a bungie cord. The contestants start at the same corner of a tennis court and walk in opposite directions along the boundary of the court. They are not allowed to backtrack along their path. The court is a rectangle, 10 meters x 20 meters. There is a electronic device attached to the midpoint of the bungie chord, and in the middle of the court is a 1m x 1m sensor (centered 5 m from the short side and 10 m from the long side) that will detect if the device is directly above it. The contestants win if they can walk around the court without the sensor detecting the device. Is it possible for them to win the game? If so, how? If not, why not?

Solution: Let $A$ and $B$ be the points on the boundary representing the 2 players, and $C$ be the point in the center of the rectangle. The angle $ACB$ starts at 0 and goes to $2\pi$ as the players walk around the court in their respective directions. By the intermediate value theorem and since the players must walk in a continuous manner, there is some point which the angle is $\pi$, that is the players and the center of the court are on a straight line. It is easy to verify using similar triangles, that any straight line through the center of a rectangle is bisected by the center. Thus the game is impossible to win (if the players don’t manually stretch the bungie toward one of the players by pulling on it.)

Alternate solution: The players must stay on the boundary, so the set of possible midpoints can only be given by the average of points from certain pairs of the sets

$$\{(x, y) : x = 0, y \in [0, 10]\}, \{(x, y) : x = 20, y \in [0, 10]\}, \{(x, y) : x \in [0, 20], y = 0\},$$

and

$$\{(x, y) : x \in [0, 20], y = 10\}$$

The union of the possible midpoints is given by the sets $\{(x, y) : x \in [0, 10], y \in [0, 5]\} \cup \{(x, y) : x \in [10, 20], y \in [5, 10]\}$. Geometrically this divides the rectangle into two regions that share the midpoint. As the midpoint moves from one corner to the other, the path it takes must go through the center of the rectangle to stay within the set of possible midpoints.

The Problem of the Week is open to all undergraduate students, regardless of major. Submit your written solution, along with your name and e-mail address, to the Math Department office (Founders Hall Room 2006) by 2:00 p.m. on Friday January 20, 2017. There is a prize of your choice of a $10 gift certificate to either Komal or Barista’s for the best solution.

http://www.unk.edu/academics/math/problem-of-the-fortnight.php

Disclaimer: Trivial solutions, although valid, run the risk of not being the best solution.